

לוגיקה: תחשיב הפרדיקטים

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חזרה קצרה

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סובייקטים ופרדיקטים

בתחשיב הפרדיקטים
כל פסוק אטומי מורכב מ
פרדיקט אחד
"סובייקט" אחד או יותר

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שני הכמתים של תחשיב הפרדיקטים

| סימן | ביטוי | שם |
|-----------|----------------|-------------------------|
| \forall | כל, כולם | כמת כולל (אוניברסלי) |
| \exists | כמה, לפחות אחד | כמת יישי |

\forall הפוך 'A'

\exists הפוך 'E'

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כמתים ספציפיים - סיכום

| | | | |
|--------------------------|---------------------|--------------------|-------------------------------------|
| כולם/ן H | $\forall x Hx$ | כל F היא H | $\forall x(Fx \rightarrow Hx)$ |
| לא כולם/ן H | $\sim \forall x Hx$ | לא כל F היא H | $\sim \forall x(Fx \rightarrow Hx)$ |
| כולם/ן לא-H | $\forall x \sim Hx$ | כל F היא לא-H | $\forall x(Fx \rightarrow \sim Hx)$ |
| יש מישהו/י שהוא/היא H | $\exists x Hx$ | יש F שהוא/היא H | $\exists x(Fx \& Hx)$ |
| אף אחד/ת אינו/ה H | $\sim \exists x Hx$ | אף F אינו/ה H | $\sim \exists x(Fx \& Hx)$ |
| יש מישהו/י שהוא/היא לא-H | $\exists x \sim Hx$ | יש F שהוא/היא לא-H | $\exists x(Fx \& \sim Hx)$ |

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אסטרטגיה כללית

- (1) נספור את מספר הכמתים במשפט
- (2) נזהה את המבנה הכללי של המשפט
- (3) נעבוד על רכיבים בנפרד
- (4) נחזיר כל רכיב לנוסחה הכללית
- (5) נספור את מספר הכמתים שהתקבלו בנוסחה הכללית
- (6) נשווה את (5) ל- (1)

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פרק 7 פרדיקטים פוליאדים

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דוגמאות של פרדיקטים פוליאדים

| | | |
|---------|----------|---------|
| Jay | respects | Kay |
| Jay | respects | some F |
| Jay | respects | every F |
| some F | respects | Kay |
| every F | respects | Kay |
| some F | respects | some G |
| some F | respects | every G |
| every F | respects | some G |
| every F | respects | every G |

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דוגמה 5

| | |
|--|--------------------------------------|
| someone respects someone | |
| there is someone | who respects someone |
| there is some x : x respects someone | |
| $\exists x$ | x respects someone |
| | there is someone whom x respects |
| | there is some y : x respects y |
| | $\exists y Rxy$ |
| $\exists x \exists y Rxy$ | |

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דוגמה 6

| | |
|---|--|
| no one respects everyone | |
| there is no one | who respects everyone |
| there is no x : x respects everyone | |
| $\sim \exists x$ | x respects everyone |
| | no matter who you are x respects you |
| | no matter who y is x respects y |
| | $\forall y Rxy$ |
| $\sim \exists x \forall y Rxy$ | |

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דוגמה 7

| | |
|---|---------------------------------------|
| no one is respected by everyone | |
| there is no one | who is R'ed by everyone |
| there is no x : x is R'ed by everyone | |
| | everyone respects x |
| $\sim \exists x$ | everyone respects x |
| | no matter who you are you respect x |
| | no matter who y is y respects x |
| | $\forall y Ryx$ |
| $\sim \exists x \forall y Ryx$ | |

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חומר חדש

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כפל משמעות

כולם מכבדים מישהו
everyone respects someone

כפל משמעות

everyone respects someone **or other**
vs.
everyone respects someone (**the same one**)
=
there is someone whom everyone respects

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מה ההבדל?

מקרה 1

does everyone respect someone? YES

is there someone whom everyone respects? NO

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מה ההבדל? (במשך)

מקרה 2

does everyone respects someone? YES

is there someone whom everyone respects? YES

NOTE: the latter implies the former

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דוגמה 8

| | |
|--------------------------------------|---------------------|
| everyone respects someone (or other) | |
| no matter who you are | you respect someone |
| no matter who x is | x respects someone |

| | | |
|-------------|-------------------|---------------------------|
| $\forall x$ | | x respects someone |
| | there is someone | whom x respects |
| | there is some y : | x respects y |
| | | $\exists y Rxy$ |
| | | $\forall x \exists y Rxy$ |

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דוגמה 9

| | |
|--------------------------------------|------------------------|
| everyone respects someone (the same) | |
| there is some one | whom everyone respects |
| there is some x : | everyone respects x |

| | | |
|-------------|-----------------------|---------------------------|
| $\exists x$ | | everyone respects x |
| | no matter who you are | you respect x |
| | no matter who y is | y respects x |
| | | $\forall y Ryx$ |
| | | $\exists x \forall y Ryx$ |

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דוגמה 10

| | | |
|-------------------|--|---------------------|
| there is some one | | who respects no one |
| there is some x : | | x respects no one |

| | | |
|-------------|-----------------|--------------------------------|
| $\exists x$ | | x respects no one |
| | there is no one | whom x respects |
| | there is no y : | x respects y |
| | | $\sim \exists y Rxy$ |
| | | $\exists x \sim \exists y Rxy$ |

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דוגמה 11

| | | |
|------------------------|---------------------|----------------------|
| | there is some one | whom no one respects |
| | there is some x : | no one respects x |
| ∃x | no one respects x | |
| | there is no one | who respects x |
| | there is no y : | y respects x |
| | ~∃y R _{yx} | |
| ∃x ~∃y R _{yx} | | |

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דוגמה 12

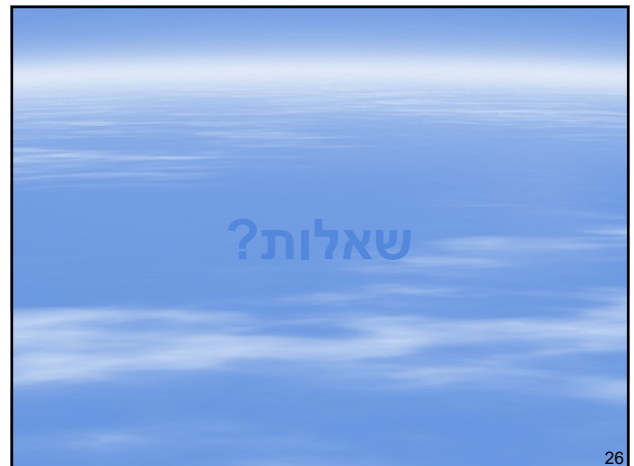
| | |
|----------------------------|--------------------------|
| Jay doesn't respect anyone | |
| no matter who you are | Jay does not respect you |
| no matter who x is | Jay does not respect x |
| ∀x | Jay does not respect x |
| | ~R _{jx} |
| ∀x ~R _{jx} | |

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סיכום – 2 כמתים, פרדיקט 1

| | |
|---------------------------------------|------------------------|
| everyone respects everyone | ∀x ∀y R _{xy} |
| someone respects someone | ∃x ∃y R _{xy} |
| no-one respects everyone | ~∃x ∀y R _{xy} |
| no-one is respected by everyone | ~∃x ∀y R _{yx} |
| everyone respects someone (or other) | ∀x ∃y R _{xy} |
| there is someone whom everyone R's | ∃x ∀y R _{yx} |
| there is someone who respects no-one | ∃x ~∃y R _{xy} |
| there is someone whom no-one respects | ∃x ~∃y R _{yx} |

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דוגמה 1 (כמת 1, 2 פרדיקטים)

| | | | |
|-----------------------------|------------------|------------------|-------------------|
| some Student Respects Jay | | | |
| there is some one | who is a student | | |
| | AND | who respects Jay | |
| there is some x | x is a student | | |
| | AND | x respects Jay | |
| ∃x | (Sx | & | R _{xj}) |
| ∃x (Sx & R _{xj}) | | | |

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דוגמה 2 (כמת 1, 2 פרדיקטים)

| | | | |
|-----------------------------|------|-------------------|-------------------|
| every Student Respects Kay | | | |
| no matter who you are | IF | you are a student | |
| | THEN | you respect Kay | |
| no matter who x is | IF | x is a student | |
| | THEN | x respects Kay | |
| ∀x | (Sx | → | R _{xk}) |
| ∀x (Sx → R _{xk}) | | | |

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דוגמה 3 (כמת 1, 2 פרדיקטים)

| | | | |
|---|------------------|-------------------|-------|
| Jay R espects no S tudent | | | |
| there is no one | who is a student | | |
| | AND | whom Jay respects | |
| there is no x | x is a student | | |
| | AND | Jay respects x | |
| $\sim\exists x$ | (Sx | & | Rjx) |
| $\sim\exists x (Sx \& Rjx)$ | | | |

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דוגמה 4 (2 כמתים, 2 פרדיקטים)

| | | | |
|---|------|---------------------|----------------------|
| every S tudent R espects someone or other | | | |
| no matter who you are | IF | you are a student | |
| | THEN | you respect someone | |
| no matter who x is | IF | x is a student | |
| | THEN | x respects someone | |
| $\forall x$ | (Sx | → | x respects someone) |
| $\forall x (Sx \rightarrow x \text{ respects someone})$ | | | |

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דוגמה 4 (ב)

| | | |
|--|-----------------------|---|
| $\forall x (Sx \rightarrow$ | x R 's someone |) |
| there is someone | whom x R 's | |
| there is some y | x R 's y | |
| $\exists y$ | Rxy | |
| $\forall x (Sx \rightarrow \exists y Rxy)$ | | |

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דוגמה 5 (2 כמתים, 2 פרדיקטים)

| | |
|---|-------------------------------|
| no-one R espects every P olitician | |
| there is no one | who respects every politician |
| there is no x : | x respects every politician |
| $\sim\exists x$ | |
| $\sim\exists x (x \text{ respects every politician})$ | |

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דוגמה 5 (ב)

| | |
|--|---|
| $\sim\exists x$ | x R espects every P olitician |
| no matter who you are | IF you are P |
| | THEN x R 's you |
| no matter who y is | IF y is P |
| | THEN x R 's y |
| $\forall y$ | (Py → Rxy) |
| $\sim\exists x \forall y (Py \rightarrow Rxy)$ | |

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