לוגיקה:
תחשיב הפסוקים

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חברי.ות הסגל שלום,

בימים אלו מופץ סקר שביעות רצון מההוראה בקרב הסטודנטים.יות. הסקר משמעותי וחשוב משום שהוא מאפשר לנו המרצים.ות להקשיב לקולם.ן של הסטודנטים.יות ולהבין את החוויות והצרכים שלהם.ן.

אנא הקצו מספר דקות במהלך השיעור ואפשרו לסטודנטים.יות לענות על הסקר.

הסקר מופץ בין התאריכים 1.1.2023-21.1.2023
חשוב לציין בפניהם.ן שהסקר אנונימי והמרצה רואה את הממצאים רק לאחר תקופת הבחינות.

להלן קישור לסקר אותו תוכלו להציג או לשלוח לסטודנטים.יות ולעודד את השתתפותם.ן

הקישור לסקר מופיע בעמוד הראשי של המודל.

קישור לסקר

בתודה מראש על שיתוף הפעולה!
היחידה לקידום איכות ההוראה והלמידה

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יחידה 2
הוכחות בתחשיב הפסוקים

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הרעיון הבסיסי

טבלאות אמת הן כלי לא מאד יעיל
אנחנו מפרטים את **כל** המקרים האפשריים
בעוד ש**רק** המקרים בהם **כל** ההנחות אמיתיות מעניינים אותנו.

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Basic Idea

We start with a few argument forms,
which we **know** are valid,
and we use these
to demonstrate that other argument forms are valid.

We demonstrate (show) that
a given argument form is valid by
deriving (**deducing**) its conclusion
from its premises
using a few **fundamental modes of reasoning**.

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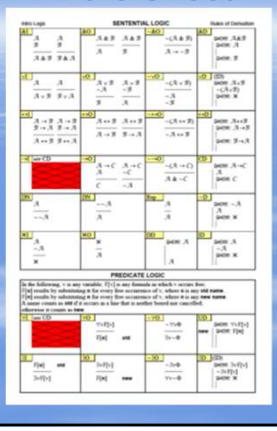
Rule Sheet

provided on exam

available on Moodle

make a copy and

keep it in front of you when doing homework



don't make up your own rules

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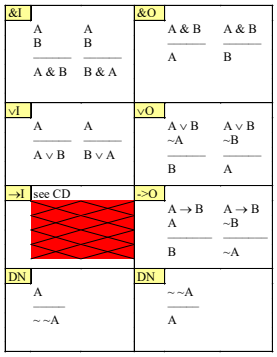
Inference Rules (so far)



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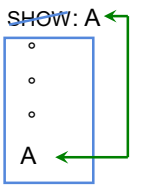
Direct Derivation

The Original and Fundamental ~~SHOW~~-Rule



In Direct Derivation (**DD**), one ***directly*** arrives at the **very** formula one is trying to show.

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Arrow-Out Strategy

If you ***have*** a line of the form A → C,
then try to apply **arrow-out** (→O),
which requires a second formula as input,
in particular, either A or ~C.

have	A→C		
find	A	**or**	~C
deduce	C		~A

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Wedge-Out Strategy

If you ***have*** a line of the form AvB,
then try to apply **wedge-out** (vO),
which requires a second formula as input,
in particular, either ~A or ~B.

have	AvB		
find	~A	**or**	~B
deduce	B		A

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Example 3

~S ; R→S ; R v ~T ; ~P v T ; ~P→~Q / ~Q

(1)	~S	Pr	
(2)	R → S	Pr	
(3)	R v ~T	Pr	
(4)	~P v T	Pr	
(5)	~P → ~Q	Pr	
(6)	~~SHOW~~: ~Q	DD	
(7)	~R	1,2,	→O
(8)	~T	3,7,	vO
(9)	~P	4,8,	vO
(10)	~Q	5,9,	→O

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Example 4

(P → Q) ∨ R ; [(P → Q) ∨ R] → ~R ;
(P → Q) → (Q → R) / ~Q

(1)	(P → Q) ∨ R	Pr	
(2)	[(P → Q) ∨ R] → ~R	Pr	
(3)	(P → Q) → (Q → R)	Pr	
(4)	~~SHOW~~: ~Q	DD	
(5)	~R	1,2,	→O
(6)	P → Q	1,5,	∨O
(7)	Q → R	3,6,	→O
(8)	~Q	5,7,	→O

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Wedge-In Strategy

if you need

AvB

then look for either disjunct:

find A

OR

find B

then

apply vI

to get AvB

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Example 5

(P v Q) → S ; (R v S) → T ; P / T

(1)	(P v Q) → S	Pr	
(2)	(R v S) → T	Pr	
(3)	P	Pr	
(4)	SHOW: T	DD	
(5)	P v Q	3,	vI
(6)	S	1,5,	→O
(7)	R v S	6,	vI
(8)	T	2,7,	→O

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Ampersand-Out Strategy

if you ***have*** a line of the form

A&B,

then apply **ampersand-out** (&O),

which can be applied immediately to produce

A

and

B

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Ampersand-In Strategy

if you are trying to find or show

A&B

then look for both conjuncts:

find A

AND

find B

then

apply &I

to get A&B

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Example 6

P → Q ; R v (Q → S) ; R → T ; ~T & P / Q & S

(1)	P → Q	Pr	
(2)	R v (Q → S)	Pr	
(3)	R → T	Pr	
(4)	~T & P	Pr	
(5)	SHOW:Q & S	DD	
(6)	~T	4,&O	
(7)	P		
(8)	Q	1,7,	→O
(9)	~R	3,6,	→O
(10)	Q → S	2,9,	vO
(11)	S	8,10,	→O
(12)	Q & S	8,11,	&I

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Can we show the following using DD?

(1) P → Q Pr

(2) Q → R Pr

(3) SHOW: P → R ??

we are stuck!!

☹

we have	P→Q	we also have	Q→R
so to apply	→O	so to apply	→O
we must find	P	we must find	Q
or find	~Q	or find	~R

Arrow-Out Strategy

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Conditional Derivation (CD)

~~SHOW: A→C~~ CD
A As
~~SHOW: C~~ ?D
°
°
°

conditional derivation
assumption
depends on formula

Affiliated Assumption-Rule

if one has a line of the form SHOW: A→C
then one is entitled to write the formula A
on the ***very next*** line, as an assumption

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Example 1

(1)	P → Q	Pr	
(2)	Q → R	Pr	
(3)	~~SHOW: P → R~~	CD	
(4)	P	As	
(5)	~~SHOW: R~~	DD	
(6)	Q	1,4,	→O
(7)	R	2,6,	→O

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Example 2

(1)	P → Q	Pr	
(2)	(P&Q) → R	Pr	
(3)	~~SHOW: P → R~~	CD	
(4)	P	As	
(5)	~~SHOW: R~~	DD	
(6)	Q	1,4,	→O
(7)	P&Q	4,6,	&I
(8)	R	2,7,	→O

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Example 3

(1)	P → (Q → R)	Pr	
(2)	~~SHOW: (P → Q) → (P → R)~~	CD	
(3)	P → Q	As	
(4)	~~SHOW: P → R~~	CD	
(5)	P	As	
(6)	~~SHOW: R~~	DD	
(7)	Q → R	1,5,	→O
(8)	Q	3,5,	→O
(9)	R	7,8,	→O

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Can we show the following using DD?

(1) P → Q Pr
(2) Q → ~P Pr
(3) SHOW: ~P ??

We are stuck!!

we have	P→Q	we also have	Q→~P
so to apply	→O	so to apply	→O
we must find	P	we must find	Q
or find	~Q	or find	~~P

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Negation Derivation (~D)

~~SHOW: ~A~~ ~D
A As
~~SHOW: X~~ DD
°
°
X XI

assumption
special symbol for "**absurdity**"
always done by DD
see later for details

to show that A is **false** (**~**A),
one **assumes** that A is **true**,
and shows that this leads to **absurdity**

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Fancy Name

Historically, this method of reasoning is called
REDUCTIO AD ABSURDUM
(Latin for "reduction to absurdity")

to demonstrate that a proposition is **false**,
one **assumes** that it is **true**,
and demonstrates that
this assumption leads to **absurdity**

in symbolic logic, an **absurdity** is a
self-contradiction –
both asserting and denying the same proposition.

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דוגמה מפורסמת: $\sqrt{2}$ אינו רציונלי

1. נניח ש- $\sqrt{2}$ הוא מספר רציונאלי (reductio הנחה לצורך)
2. $\sqrt{2} = \frac{n}{m}$ כאשר n ו- m שלמים זרים כלשהם
3. $2 = (\frac{n}{m})^2$
4. $2m^2 = n^2$
5. לכן, n זוגי (n=2k)
6. $2m^2 = 4k^2$
7. $m^2 = 2k^2$
8. לכן, m זוגי
9. לכן, n ו- m אינם שלמים זרים
10. סתירה (שורות 2 ו- 9) – לכן 1 שקרי.

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Affiliated Rules

Contradiction-In (XI)

if you have a formula	A
and you have its negation	~A
then you are entitled to infer	———
a contradiction (absurdity)	X

Assumption Rule

If one has a line of the form	SHOW: ~A
then one is entitled to write the formula	A
on the *very next* line, as an assumption.	

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Example 4

(1)	P → Q	Pr	
(2)	Q → ~P	Pr	
(3)	SHOW: ~P	~D	
(4)	P	As	
(5)	SHOW: X	DD	
(6)	Q	1,4,	→O
(7)	~P	2,6,	→O
(8)	X	4,7,	XI

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