לוגיקה:
תחשיב הפסוקים

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יחידה 2
הוכחות בתחשיב הפסוקים

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הרעיון הבסיסי

טבלאות אמת הן כלי לא מאד יעיל
אנחנו מפרטים את **כל** המקרים האפשריים
בעוד ש**רק** המקרים בהם **כל** ההנחות אמיתיות מעניינים אותנו.

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Basic Idea

We start with a few argument forms,
which we **know** are valid,
and we use these
to demonstrate that other argument forms are valid.

We demonstrate (show) that
a given argument form is valid by
deriving (**deducing**) its conclusion
from its premises
using a few **fundamental modes of reasoning**.

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Example 1 – Modus Ponens (MP)

A → C
A
———
C

if A then C
A
———
C

a *derivative* argument form

we can employ
modus ponens (MP)
to *derive* the conclusion from
the premises.

P
P → Q
Q → R
R → S
———
S

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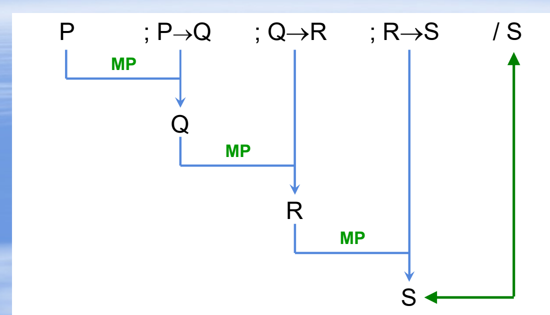
Example 1 (continued)



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Derivations – How to Start

argument : P ; P→Q ; Q→R ; R→S / S

1. write down premises
2. write down "SHOW:" conclusion

(1)	P	Pr(emise)
(2)	P → Q	Pr
(3)	Q → R	Pr
(4)	R → S	Pr
(5)	**SHOW**: S	**(the goal)**

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Derivations – How to Continue

3. apply rules, as appropriate, to *available* lines until goal is reached

(1)	P	Pr
(2)	P → Q	Pr
(3)	Q → R	Pr
(4)	R → S	Pr
(5)	SHOW: S	**(goal)**
(6)	Q	1,2, MP
(7)	R	3,6, MP
(8)	S	4,7, MP

follows from lines 1 and 2 by modus ponens

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Derivations – How to Finish

4. Box and Cancel

(1)	P	Pr
(2)	P → Q	Pr
(3)	Q → R	Pr
(4)	R → S	Pr
(5)	SHOW: S	DD
(6)	Q	1,2, MP
(7)	R	3,6, MP
(8)	S	4,7, MP

DD = Direct Derivation

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Example 2

~S ; R → S ; Q → R ; P → Q ; / ~P

(1)	~S	Pr
(2)	R → S	Pr
(3)	Q → R	Pr
(4)	P → Q	Pr
(5)	SHOW: ~P	DD
(6)	~R	1,2, MT
(7)	~Q	3,6, MT
(8)	~P	4,7, MT

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Example 3

~S ; R→S ; ~R→~T ; P→T ; ~P→~Q / ~Q

(1)	~S	Pr
(2)	R → S	Pr
(3)	~R → ~T	Pr
(4)	P → T	Pr
(5)	~P → ~Q	Pr
(6)	SHOW: ~Q	DD
(7)	~R	1,2, MT
(8)	~T	3,7, MP
(9)	~P	4,8, MT
(10)	~Q	5,9, MP

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Initial Inference Rules

Modus Ponens

A → C
A
———
C

Modus Tollens

A → C
~C
———
~A

Modus Tollendo Ponens (1)

A v B
~A
———
B

Modus Tollendo Ponens (2)

A v B
~B
———
A

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Examples of Modus Ponens

A → C A ——— C	(P&Q) → (RvS) P&Q ——— RvS	~P → ~Q ~P ——— ~Q

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Examples of Modus Tollens

A → C ~C ——— ~A	(P&Q) → (RvS) ~(RvS) ——— ~(P&Q)	~P → ~Q ~~Q ——— ~~P

~P → ~Q
Q
———
P

valid argument

BUT

NOT an example of **MT**

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Form versus Content/Value

are the dime and ten pennies the same? NO

are they the same **in value**? YES

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Examples of MTP(1)

A v B ~A ——— B	(P&Q) v (RvS) ~(P&Q) ——— RvS	~P v ~Q ~~P ——— ~Q

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Examples of MTP(2)

A v B ~B ——— A	(P&Q) v (RvS) ~(RvS) ——— P&Q	~P v ~Q ~~Q ——— ~P

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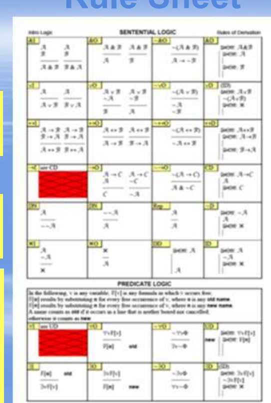
Rule Sheet

provided on exam

available on Moodle

make a copy and

keep it in front of you when doing homework



don't make up your own rules

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Rules of Inference – Basic Idea

(almost) every connective has an

- **OUT**-rule — how to **break-down** a formula with this connective
- **IN**-rule — how to **build-up** a formula with this connective

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Rules discussed next

&I A, B / ——— / A & B A, B / ——— / B & A	&O A & B / ——— / A A & B / ——— / B
∨I A / ——— / A ∨ B A / ——— / B ∨ A	∨O A ∨ B, ~A / ——— / B A ∨ B, ~B / ——— / A
→I see CD	->O A → B, A / ——— / B A → B, ~B / ——— / ~A
DN A / ——— / ~ ~A	DN ~ ~A / ——— / A

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Ampersand-Out (&O)

if you have a conjunction then you are entitled to infer its first conjunct	A & B ——— A
if you have a conjunction then you are entitled to infer its second conjunct	A & B ——— B

'have' means 'have as a **whole** line'

rules apply **only** to whole lines
not pieces of lines

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Ampersand-IN (&I)

if you have a formula and you have a formula then you are entitled to infer their (first) conjunction	A B ——— A & B
if you have a formula and you have a formula then you are entitled to infer their (second) conjunction	A B ——— B & A

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Wedge-Out (vO)

if you have a **disjunction** **and** you have the negation of its 1st disjunct then you are entitled to infer its second disjunct	A v B ~A ——— B
if you have a **disjunction** **and** you have the negation of its 2nd disjunct then you are entitled to infer its first disjunct	A v B ~B ——— A

what we earlier called 'modus tollendo ponens'

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Wedge-IN (vI)

if you have a formula then you are entitled to infer its disjunction with ***any*** formula to its right	A ——— A **v** B
if you have a formula then you are entitled to infer its disjunction with ***any*** formula to its left	A ——— B **v** A

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Arrow-Out (→O)

if you have a **conditional**	A → C
and you have its antecedent	A
then you are entitled to infer	———
its consequent	C

if you have a **conditional**	A → C
and you have the negation of its consequent	~C
then you are entitled to infer	———
the negation of its antecedent	~A

what we earlier called
'modus ponens' and 'modus tollens'

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Arrow-Introduction

THERE IS NO SUCH THING AS ARROW-IN (→I)

☠

what we have instead is
CONDITIONAL DERIVATION
(CD)

☺

which we examine later

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Double-Negation (DN)

if you have a formula	A
then you are entitled to infer	———
its double-negative	~~A

if you have a double-negative	~~A
then you are entitled to infer	———
the formula	A

rules apply **only** to whole lines
not pieces of lines

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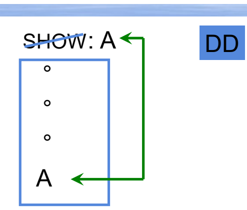
Direct Derivation

The Original and Fundamental ~~SHOW~~-Rule



In Direct Derivation (**DD**),
one ***directly*** arrives at
the **very** formula
one is trying to show.

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Arrow-Out Strategy

If you ***have*** a line of the form A → C,
then try to apply **arrow-out** (→O),
which requires a second formula as input,
in particular, either A or ~C.

have	A→C		
find	A	**or**	~C
deduce	C		~A

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